Ergodic Random Process

- How to estimate parameter of the random process from measurement data?
- Time average of random process X(t)

$$\bar{x} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\,dt$$

- Expected value $E[X(t)]$,also called ensemble average
- Ergodic random process is a stationary process in which every sample function exhibits the same statistical behaviour as ensemble
- It is possible to determine the statistical behaviour of the ensemble by examining only one sample function

Ergodic Random Process

- For an ergodic random process, mean values and moments can be determined by time averages.

$$E[X^n] = \overline{X}^n = \int_{-\infty}^{\infty} x^n f_X(x)dx = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} X^n(t)dt$$

- X(t) is mean ergodic(or ergodic in the mean) if

$$E[X(t)] = \overline{x}$$

Ergodic Random Process

- **Example**
- Random process $X(t) = A\cos(wt + \Theta)$
- A: r.v +1,-1 with equal prob.
- Θ: uniformly distributed between 0 and 2π
- Assume A and Θ are independent
- X(t) : mean ergodic?

Ergodic Random Process

- **Example (cont)**

$$\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}X(t)dt=\lim_{T\to\infty}\frac{1}{2T}\int_{0}^{2\pi}A\cos(wt+\Theta)dt$$

$$=\lim_{T\to\infty}\frac{A}{2Tw}[\sin(wt+\Theta)]_0^{2\pi}=\lim_{T\to\infty}\frac{A}{2Tw}[\sin(2\pi w+\Theta)-\sin\Theta]=0$$

$$E[X(t)]=E[A\cos(wt+\Theta)]=E[A]E[\cos(wt+\Theta]=0$$

- X(t) is mean ergodic